Dynamic Bulk Moduli of Several Solids Impacted by Weak Shockwaves

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The bulk compressibility of several solids was measured using attenuated underwater shockwaves. Attenuation was such as to deliver an approximate sonic pulse to the test specimens. Measurements were made of the wave-attenuation and the wave-transit time in several metals, plastics, and chemical compounds. High-speed smear-camera shadowgraphs were used for the measurements.

INTRODUCTION

For a shockwave moving through material initially in a state with pressure P_0, specific volume V_0, and specific internal energy E_0, the equations conserving mass, momentum, and energy are

$$P - P_0 = (1/V_0)U_s u_p, \tag{1}$$

$$V_0/V = U_s/(U_s - u_p), \tag{2}$$

and

$$E - E_0 = \tfrac{1}{2}(P + P_0)(V_0 - V), \tag{3}$$

where P, E, and V are the pressure, energy, and specific volume; U_s is the velocity of the shockwave; and u_p is the particle velocity of the material behind the shock front. An analysis of the P, V, and E data in the compressed state is provided by the solution of Eq. 3, which gives as a projection in the P–V plane the so-called Hugoniot curve. Experimentally, however, one usually measures the parameters in the U_s–u_p plane and the Hugoniot curve is represented there.

For this representation, many investigations[1] have shown (in the absence of shock-induced phase changes) that the data in the U_s–u_p plane are best represented by a straight line, i.e.,

$$U_s = C_0 + S u_p. \tag{4}$$

Eliminating u_p in Eqs. 1 and 2 gives

$$U_s^2 = V_0^2 (P - P_0)/(V_0 - V), \tag{5}$$

and the intercept C_0 in the U_s–u_p plane described by Eq. 4 is the limiting value of U_s as $P \to P_0$ and

[1] R. G. McQueen and S. P. Marsh, J. Appl. Phys. **31**, 1253–1269 (1960).

$V \to V_0$, i.e.,

$$C_0^2 = -V_0^2 (dP/dV)\big|_{P=0}. \tag{6}$$

The connection between C_0 and plane elastic wave compression is readily apparent. Consider the two types of waves that can be propagated in an isotropic solid, namely, longitudinal waves with their particle motion in the direction of the propagation and shear waves with their particle motion transverse to the direction of propagation. These waves have the respective velocities

$$C_l^2 = (\lambda + 2\mu)/\rho \tag{7}$$

and

$$C_s^2 = \mu/\rho, \tag{8}$$

where μ and λ are the two Lamé elastic modulii and ρ is the density $(1/V)$. For a plane-wave compression of the isotropic solid, the strains

$$\epsilon_{11} = \epsilon_{22} = \epsilon_{33} = \tfrac{1}{3}\Delta V/V. \tag{9}$$

Using Eq. 9 and the fact that the modulus of bulk compressibility is the ratio of the hydrostatic pressure P to the compression $\Delta V/V$ allows the modulus of bulk compressibility to be expressed as

$$\frac{P}{\Delta V/V} = \frac{3\lambda + 2\mu}{3} = \lambda + \tfrac{2}{3}\mu \tag{10}$$

and

$$P = (\lambda + \tfrac{2}{3}\mu)[(V_0 - V)/V]. \tag{11}$$

Therefore,

$$C_0^2 = (\lambda + \tfrac{2}{3}\mu)/\rho_0, \tag{12}$$

and the intercept for zero particle velocity is the bulk

TABLE I. Epoxy resin composition and curing condition.

Epoxy resin	Composition[a] (parts by weight)	Curing condition
Epoxy-1	50/EPON 826; 50/DER 732; 9/curing agent A	72 h at 23°C +1 h at 115°C
Epoxy-2	100/EPON 828; 14.5/curing agent CL	1 h at 115°C
Epoxy-3	70/EPON 826; 30/DER 732	18 h at 23°C +2 h at 100°C
Epoxy-4	50/EPON 826; 50/DER 732; 9/curing agent A	24 h at 23°C +2 h at 100°C

[a] Compositions are commercial designations (Shell Chemical Co.). The epoxides were polymerized with stoichiometric amounts of amine curing agents, e.g., m-phenylenediamine.

sound speed. Its value in terms of the Lamé constants is obtained by using the relations for C_l and C_s:

$$C_0^2 = \frac{\lambda + \frac{2}{3}\mu}{\rho_0} = \frac{\lambda + 2\mu}{\rho_0} - \frac{4}{3}\frac{\mu}{\rho_0} \quad (13)$$

and

$$C_0^2 = C_l^2 - \frac{4}{3}C_s^2. \quad (14)$$

We note that the ultrasonic bulk sound speed is not measured directly by static means but is obtained from measurements of the longitudinal and shear sound velocities through Eq. 14.

From Eqs. 1, 2, and 4 the experimental dynamic pressures are given by

$$P = C_0^2(V_0 - V)/[V_0 - S(V_0 - V)]^2, \quad (15)$$

and differentiation yields the dynamic bulk modulus

$$-V\frac{dP}{dV} = -C_0^2 \frac{(V/V_0)\{1 + S[1 - (V/V_0)]\}}{V_0\{1 - S[1 - (V/V_0)]\}^3} \quad (16)$$

Thus, accurate estimates of C_0 and S from shock measurements permit a useful extension of dynamic compression data into low-pressure regions, which normally are accessible only to static techniques. This allows an inspection of the relationship between dynamic and ultrasonic determinations of the bulk modulus.

To determine S accurately requires extensive shock-wave measurements, although a value of S for a linear U_s-u_p relation can be calculated[2] from knowledge of the thermodynamic Grüneisen constant γ (evaluated at $P=0$) and the coefficient of volume expansion.

In the absence of these data, we have developed an optical technique,[3] using the attenuation of weak shock waves in water to deliver an approximate sonic pulse for obtaining the lower limit of shock-propagation data. This method has been used to estimate the bulk sound speeds and the isentropic bulk modulii of various solids,

[2] J. Berger and S. Joigneau, Compt. Rend. **249**, 2506–2508 (1959). See also D. Pastine and D. Piacesi, J. Phys. Chem. Solids, **27**, 1783–1792 (1966).

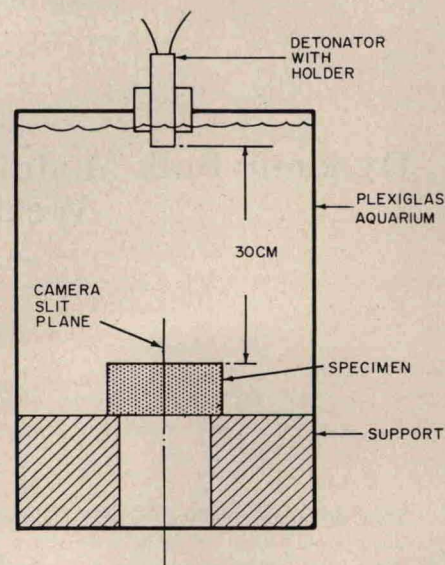[3] J. M. Majowicz, Naval Ordnance Laboratory (unpublished rep.).

FIG. 1. Experimental assembly for initiating and measuring weak underwater shocks.

including propellants and explosives, to which conventional static or ultrasonic methods are often difficult to apply. The method also was used to examine the compressive behavior of several cross-linked polymers (Table I) based upon epoxide resins.[4] This paper reports these measurements and gives results for several metals, chemical compounds, and more common polymer materials.

I. EXPERIMENTAL

In the arrangement shown in Fig. 1, a cylinder or slab of a specimen material is immersed in a transparent liquid, usually water contained in a Plexiglas aquarium. The specimens range in thickness from 1.27 to 5.1 cm and are positioned with their plane–parallel surfaces perpendicular to the direction of the incoming wave. An electric detonator (or exploding tungsten wire) is centered in the liquid, 30 cm from the plane surface of the specimen. The initiation of the detonator produces a very weak shock wave, which is nearly planar when it arrives, $\sim 200\,\mu$sec later, at the liquid–specimen interface.

The propagation of this incident wave normal to the specimen's parallel surface and the motion of the emerging wave transmitted by the specimen to the liquid below are recorded by a rotating-mirror smear camera along a central narrow line defined by the camera slit. The wave propagation is made visible by "back-lighting" using a shadowgraph technique.[5] In

[4] These resins are Epon 828 and Epon 326 from Shell Chemical Company and are essentially diglycidyl ethers of bisphenol A, with epoxide equivalents within the range of 185 to 205. See H. Lee and K. Nevill, *Epoxy Resins* (McGraw–Hill Book Co., New York, 1957), pp. 10–11, for details of preparation and basic unit structure.

[5] T. P. Liddiard, Jr., and B. E. Drimmer, J. Soc. Motion Picture Television Eng. **70**, 106–110 (1961).

TABLE II. Sound velocities and compressibilities.

Specimen material	ρ_0 (g/cm³)	C_0 (m/sec)	Intercept U_s (m/sec)	Longitudinal Sound velocity (m/sec)	Incident $U_s(H_2O)$ (m/sec)	Transmitted $U_s(H_2O)$ (m/sec)	K (cm²/dyn)
Brass (Cu Zn) 36 at.%Zn	8.40	3815	3860	4700[12]	1493	1479	0.08×10⁻¹¹
Aluminum	2.70	5454	5350	6420[12]	1480	1476	0.13×10⁻¹¹
Boron Nitride	2.20	2628	2831		1501	1496	0.66×10⁻¹¹
Vinylidene Chloride	1.60	2690			1473	1466	0.86×10⁻¹¹
TNT	1.61	2574	2390	2586±80[13]	1495	1463	0.94×10⁻¹¹
Plexiglas	1.18	2760	2710	2770[8] 2650[10]	1497	1492	1.11×10⁻¹¹ 1.3×10⁻¹¹ [9] 1.7×10⁻¹¹ [10] 2.5–3.0×10⁻¹¹ [11]
Nylon	1.15	2616	2300	2650[8]	1474	1480	1.27×10⁻¹¹
Epoxy-2	1.20	2535			1476	1458	1.30×10⁻¹¹
Epoxy-3	1.12	2402			1498	1463	1.55×10⁻¹¹
Epoxy-1	1.18	2254	2240		1500	1475	1.67×10⁻¹¹
Epoxy-4	1.16	2098			1490	1468	1.96×10⁻¹¹
Teflon	2.03	1453	1991	1340±47[8]	1494	1467	2.33×10⁻¹¹

this method, intense light is obtained from the explosion of a 5.1-cm-long 0.012-cm-diam tungsten wire threaded within a 1-mm-bore capillary tube. This light is collimated by a field lens positioned behind the aquarium. The shock motion is delineated by the disturbance it produces in the light that reaches the film through the smear-camera slit. The tungsten wire is exploded by the discharge of a 4-μF capacitor charged to 4000 V. The backlighting is usually delayed to occur ~150 μsec after the initiation of the incident shock from the detonator. A camera writing speed of ~1.3 mm/μsec is best for recording wave propagation in most liquids, using the above experimental geometry. Figure 2 is a typical record resulting from the experimental arrangement. With proper interpretation, it allows measuring the velocity of the incident wave striking the specimen, the transit time of the wave[6] in the specimen, and the velocity of the emerging wave in the water below.

II. RESULTS AND DISCUSSION

Table II gives the results obtained for several materials and lists the density, ρ_0; the sound velocity in the specimen, C_0; the velocity of the incident wave in the water, Incident $U_s(H_2O)$; and the velocity of the wave transmitted from the specimen to the water below, Transmitted $U_s(H_2O)$. Table II also contains values of the reciprocal of the bulk modulus, i.e., the initial compressibility, K, determined for $V=V_0$ in Eq. 16, and experimental values of the intercept of the linear U_s–u_p curve.

The combined errors produced by camera speed and magnification ratio measurements result in an error in the measured velocities not exceeding 0.3%. The velocity data listed are the average of three experiments run with each specimen material, except TNT and

[6] The wave propagation in transparent material, e.g., Plexiglas, is made visible by this technique.

polymethyl methacrylate (Plexiglas), for which five experiments each were conducted. The maximum mean velocity deviation was ±0.6%, and this is considered the maximum error to be expected in the measurement of velocity.

A temperature variation of ±2°C was possible during these determinations, principally because the experiments were conducted in bomb-proof facilities at times when temperature control was difficult to maintain. However, large errors due to temperature variations were avoided by adjusting the water temperature in the aquariums to 20°C at the time of firing.

The average values of the incident water-wave velocity and transmitted water-wave velocity in Table I are ~1490 and ~1470 m/sec, respectively. The velocity of sound[7] in distilled water at 20°C is 1481.63 m/sec. Therefore, considering the errors in measuring the weak
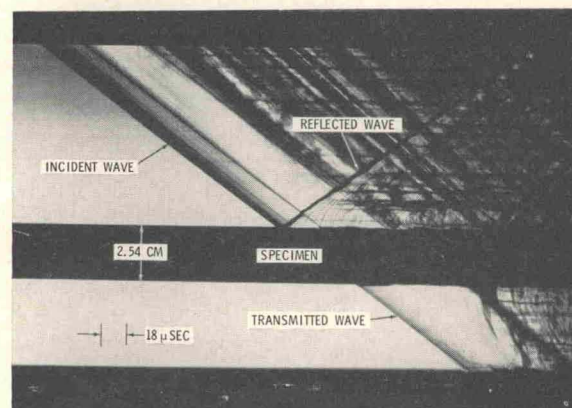

FIG. 2. Smear-camera shadowgraph of weak shock waves in water, obtained using the arrangement in Fig. 1. The upper and lower traces, respectively, are from the incident and transmitted waves in the water. The central dark bend denotes the specimen material.

[7] W. D. Wilson, J. Acoust. Soc. Amer. 31, 1067–1072 (1959).

shock propagation, the waves entering and leaving the specimen material may be treated as sonic. This view is supported by pressure measurements using piezoelectric gauges, which indicate that the incident pressure at the water–specimen interface does not exceed 50 bars.

For Plexiglas, the weak shock-velocity data compare favorably with ultrasonic measurements of longitudinal waves. For example, Auberges and Reinhart[8] give 2770 m/sec as compared to the weak shock measurement of 2760 ± 15 m/sec. From this measurement, we find $k=1.11\times10^{-11}$ cm²/dyn. Kolsky's[9] results from pressure bar experiments give $k=1.3\times10^{-11}$. Wada's[10] measurements of the velocities of longitudinal waves (2650 m/sec) and transverse waves (1240 m/sec) at 1.08×10^6 Hz give $k=1.7\times10^{-11}$ cm²/dyn. Pullen et al.[11] find much greater compressibility for molded Plexiglas specimens, with K ranging from 2.5×10^{-11} cm²/dyn to 3.0×10^{-11} cm²/dyn. These discrepancies in the compressibility and the close agreement of the weak shock measurement to the ultrasonic value for longitudinal waves in nylon, TNT, and Teflon in (Table II), indicate the method does actually give values associated with longitudinal waves. The C_0 data of Table II then would be 10%–30% greater than the bulk sound velocities calculated from the elastic constants (see Eq. 14). This would, of course, result in erroneous values of the initial dynamic compressibility in Table II, compared to static data.

However, the conditions of shock propagation are not isentropic. Dynamic compressibilities may be different from the corresponding static values obtained under equal pressures. Also, dynamic and static yield strengths of a material do not necessarily have the same value. For solids with significant shear strength, which can contribute to the material stiffness, the values of C_0 obtained by the shock-attenuation technique represent elastic-wave compressions and should agree with the velocities of low-pressure longitudinal waves. At the critical stress (the Hugoniot elastic limit), the specimen material passes from elastic compression behavior to plastic- or fluidlike flow. In reaching this point, the elastic-wave data in the shock-velocity–particle-velocity plane follow a relation with its limiting wave velocity the zero-pressure longitudinal sound speed. For the low-amplitude limit of the plastic wave, the limiting velocity is the bulk sound speed. For aluminum, however, the measured C_0 value in Table I is 5454 m/sec. This value is 15% less than the longitudinal sound velocity, (6420 m/sec), but it exceeds the ultrasonic bulk sound velocity (5375 m/sec) by only 1.5% and probably indicates the shock in aluminum attenuated to a value close to the yield stress. Similar results are obtained for brass (Cu Zn alloy containing 36 at.% Zn) as compared to the ultrasonic data available only for a Cu Zn alloy[12] containing 30 at.% Zn.

The C_0 value for Teflon, however, differs considerably ($\sim30\%$) in Table II from the intercept U_s, which was obtained by a linear extrapolation of shock data in the 25-kbar to 290-kbar range. This result indicates the $U_s–u_p$ relation for Teflon may not be linear and the possibility of a phase change exists. Several investigators including Bridgman[13] have reported that Teflon undergoes a transition at 6.5 kbar. For the other pressed materials in Table II, e.g., TNT,[14] BN,[15] and vinylidene chloride, where the incident weak shock stress approaches the yield strength, the velocities measured apparently are close to the bulk sound velocities and the compressibilities in Table II are probably accurate values.

ACKNOWLEDGMENT

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